

A normative justification of compulsory education

Alessandro Balestrino,^{*} Lisa Grazzini,[†] and Annalisa Luporini[‡]

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Abstract

Using a household production model of educational choices, we characterise a free-market situation in which some agents (high wagers) fully educate their children and spend a sizable amount of resources on them, while others (low wagers) educate them only partially. The free-market equilibrium is iniquitous, both because the households have different resources and because the children have different access to education. Public policy is thus called for, for vertical as well as horizontal equity purposes. Conventional wisdom has it that both objectives could be achieved using price control instruments, i.e. income taxes and price subsidies. We find instead that income taxes reduce equality of opportunity and that price subsidies cannot remedy this. Quantity controls become necessary: a compulsory education package, financed by a redistributive tax system, achieves both types of equity. Redistributive taxation and compulsory education are therefore best seen as complementary policies.

Keywords: Education, In-kind transfers, Redistributive taxation.

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^{*}CESifo and Department of Political Science, University of Pisa, Via Serafini 3, 56126 Pisa, Italy. E-mail: alessandro.balestrino@unipi.it

[†]Department of Economics, University of Florence, Via delle Pandette 9, 50127 Florence, Italy. E-mail: lisa.grazzini@unifi.it.

[‡]CESifo and Department of Economics, University of Florence, Via delle Pandette 9, 50127 Florence, Italy. E-mail: annalisa.luporini@unifi.it.

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I Introduction

In what sense might parents constrain rather than favour the development of their children? Mostly by under-investing in their education, a phenomenon which is by now accepted as a stylized fact in the literature. There are two competing explanations for this.

- First, there is the standard beckerian view (e.g. Becker et al. 1990) according to which parents see education as a consumption good whose enjoyment may be limited by liquidity constraints: parents are altruistic towards their children, and would like to spend as much as possible in their education, but they might be unable to afford the level of outlay which would be optimal given the potential abilities of the children. The obvious remedy for this is a redistributive policy that transfers more resources towards the needy. At least for primary and secondary education,¹ a more market-oriented solution is difficult to find, as there is no credit market for the investment in education due to the lack of collateral (future income is normally unacceptable).
- An alternative view sees education as an investment also from the standpoint of the parents and not only of the children: this perspective is related to the "exchange model" of the family pioneered e.g. by Cigno (1993). Selfish family members engage in transfers regulated by self-enforcing rules specifying rewards for obedience and punishments for deviations. The resulting system may be inefficient for several reasons, the most relevant being that parents, when investing in their children's education, foresee that they will be able to reap only a fraction of the return, and tend therefore to under-invest. Redistribution is clearly ineffective, whereas the subsidization of educational expenditure, by lowering the cost of investment, might work (Anderberg and Balestrino, 2003).

The results from the empirical literature are hardly decisive. It is true that the testable implications of the altruistic model are usually not verified (e.g. Altonji et al. 1992, 1997), whereas those of the exchange model are more consistently found to be holding (e.g. Cigno et al. 1998, 2006). It has however been argued that the test usually employed for the altruistic model is unnecessarily restrictive, and that at this stage of our general knowledge there is no definitive case in favour of one or the other approach (McGarry 2000).

¹This is not necessarily true for higher education, see for example Oei and Ring (2014) on the US case.

A point which we might want to stress is however that neither view recommends an education policy that includes, among other things, compulsory schooling.² This is in stark contrast with what actually happens in virtually all the developed countries, and has been happening for the past 150-plus years. It is a historical fact that education policy was conceived in terms of free and mandatory public schooling (financed by public funds) when it was introduced during the XIX century in the West (Germany, France and later UK and US); and free and mandatory schooling is still at the basis of our educational systems today.³ Compulsory schooling is, instead, still at stake in many less developed countries where universal primary education is far from having been achieved, especially for girls.

Economists are always suspicious of policy interventions that seem to thwart individual freedom or consumer sovereignty. It has however been recognised, at least since the contributions of Neary and Roberts (1980) and Guesnerie and Roberts (1984), that in a second-best world quantitative restrictions may be welfare-improving inasmuch as they enhance the efficiency and the redistributive impact of the tax system.⁴ While these arguments certainly pave the way for our present line of research, they are too vague for our purposes. They refer to generic commodities, and not specifically to education, a service that can of course be bought on the market as many others but has its own peculiarities. Two aspects, normally recognised in the literature on education, but not in that on rationing, are, in our opinion, worth emphasising:

1. unlike most commodities, education is purchased not by those who consume it, but by a third party (at least for primary and secondary education, the parents bear the costs of education, while the benefit will be reaped, in time, by the children);
2. the enjoyment of its fruits, no matter whether they are seen in terms of investment or consumption value, requires out-of-pocket expenses *and* a large amount of time, i.e. ample opportunity costs (education is a long process: it goes on for years).⁵

²An exception is represented by Cigno (2012) who recommends compulsory school enrolment in developing countries where child labour is an issue in order to remedy for problems deriving from asymmetric information on the use of children's time. For a comparison with our results see section IV on Alternative policy frameworks.

³For example, see Go (2009) for a paper presenting a political economy explanation for the American achievement of universal free public schooling in a historical perspective.

⁴The surveys by Balestrino (1999, 2000) illustrate the state of the art in this stream of work at the end of the '90s. For a more recent outlook, see Currie and Gahvari (2008).

⁵According to Cipolla (1969), opportunity costs appear to be the single most important factor behind the

In order to account for these peculiarities, we employ a household economics approach. We recognise that there are two actors involved in the purchase and consumption of education, the parents and the child (point 1 above) and we model time allocation in a detailed way, trying to account for its key role in the educational process (point 2). From a normative standpoint, we develop an argument showing i) that education policy is socially desirable, and ii) that it must preferably include a period of compulsory schooling rather than follow another intervention design.

The plan of the paper is the following. Section II discusses how policy objectives should be formulated. Section III presents the model of educational choice and the *laissez-faire* outcome. Section IV analyses different public policies. Finally, section V contains some concluding remarks.

II Horizontal and vertical equity in educational policy

A distinction is often made between vertical and horizontal equity. The former refers to the equalisation of resources among agents, and is normally pursued *via* progressive tax systems; the latter implies an equal treatment of equals, and requires that policies do not discriminate motivelessly. In our setting, then, vertical equity would be pursued by redistributing income among the parents, while the most important achievement in terms of horizontal equity would be that all the children are given equal access to education – what one might call "equality of opportunity". In this sense, an equitable distribution of education among the children could be judged fair.

Typically, normative economic analyses present the government's objective in terms of vertical equity and efficiency objectives. The social planner is assumed to maximise a Paretian, quasi-concave social welfare function that will pick up a point on the second-best Pareto-frontier and satisfies the vertical equity requirements implied by the convexity of the social indifference curves.

The policy approaches to education that we mentioned above fall, broadly speaking, within this framework: the optimal education policy is defined together with the optimal intra-generational

development of education systems. Thus, as long as the kid's time was valuable for help in the farm or for employment in the Dickensian factories of the early industrial revolution, large-scale education programs were not undertaken in the Western world, but became important starting from the second half of the XIX century, when productivity began to be high enough to make the kids' work dispensable.

redistribution policy as the result of social welfare maximisation. In the beckerian view, the two policies coincide, as the optimal income tax also remedies the inequality in education achievements; in the exchange view, a specific educational subsidy is needed in addition to the income tax. It is interesting to notice, however, *that the achievement of vertical equity among families does not come at the expense of equality of opportunity for the children*: income is redistributed among the parents or educational expenditure is subsidised and, at the same time, the children are all given access to education.

This outcome is in fact somewhat surprising, as it is often found that horizontal and vertical equity requirements conflict with each other (see e.g. Auerbach and Hassett 2002); however, it is predicated on models of educational choice that, in our view, do not take into full account the implications of the two peculiar characteristics of education that we highlighted before, namely the role of the parents and the relevance of the opportunity costs.

We shall see that in our model, where these two characteristics are thoroughly explored, neither the policy recommendation of the beckerian model nor that of the exchange model work as far as equality of opportunity is concerned: a different policy instrument, to wit compulsory education, is needed. This, despite the assumption of altruism within the family that we maintain throughout.

One way of viewing this is to say that, in our model, equality of opportunity for the children must be pursued with a specific quantity control, rather than with the usual armoury of price controls. In this sense, our model may be considered as an attempt to apply the now sizable literature on equality of opportunities to education.⁶ Indeed, as stated by Brunori *et al.* (2012, p. 765), ‘equality of educational opportunity is a widely agreed principle, almost universally considered to be a funding principle of education policy.’ There are of course different interpretations of the principle in the literature; in any case, even the more restrictive interpretations do not deny the importance of a minimum education level (Friedman 1962, p. 89, for example, states that ‘both the imposition of a minimum required level of schooling and the financing of this schooling by the state can be justified by the “neighborhood effects” of schooling.’)

The approach by Roemer (1998) is particularly close to our set-up. Indeed, Roemer points

⁶This literature started with Rawls (1971), and nowadays equality of opportunity can be considered ‘the prevailing conception of social justice in contemporary western societies’ (Ferreira and Peragine 2014, p. 2). Beyond the case of education, such approach has been applied to different areas of public policy such as health, anti-poverty schemes, income taxation and redistribution. For a recent survey on this topic, see Roemer and Trannoy (2015).

out that equality of opportunity can be obtained by equalizing or compensating all those individual's circumstances affecting the individual's final outcome for which she cannot be held responsible, and letting, instead, unaltered the effects of choices for which individuals are to be held responsible. Of course, this requires that it must be possible to distinguish the share of inequalities due to unequal circumstances (opportunities) from the one due to individual choices. Now, the first characteristic of (primary and perhaps secondary) education we mentioned above is that it is not chosen by its users, who cannot therefore taken to be responsible for the consequences of the choice. Then, a compensation principle can be applied to neutralize inequality in education investments.

Our position seems to capture closely what has historically happened with the introduction of compulsory education in the Western countries (see Section I above) and also the way in which current policy proposals for the developing countries are formulated, namely directly in terms of percentages of children who have access to primary education. For example, at the World Education Forum in Dakar, 2000, Goal 2 was stated as: "Ensuring that by 2015 all children, particularly girls, children in difficult circumstances and those belonging to ethnic minorities, have access to, and complete, free and compulsory primary education of good quality". The same objective has been re-affirmed again by the UNDP with the Sustainable Development Goals: Goal 4 calls for achieving inclusive and quality education for all, and more specifically it "ensures that all girls and boys complete free primary and secondary schooling by 2030." No mention, in these statements, is made of the parents or of the tax system: it is said that 100% of the children must receive *compulsory* education, which means that this specific policy instrument is called for, and no other. In other words, policy makers in the past, as well as today, appear to have pursued and to pursue horizontal equity objectives alongside the more commonly assumed vertical equity/efficiency objectives, and appear to have believed and to believe that the former require a dedicated instrument. In this paper, we will see how these prescriptions follow naturally from our model of educational choice.

III A model of educational choice

We consider a finite-horizon model.⁷ The economy is made of two-person households: one parent and one child. We posit that, in order to earn an income, each parent supplies a certain amount of labour l to the market at a wage rate denoted by w ; w varies continuously on $[0, \bar{w}]$ according to a density function $F(w)$, and the agents have unit mass.

Income can be spent on the parent's own consumption C and the child's education e (we normalise the child's consumption to zero).⁸ Education also requires time d (of the child): in fact, we attribute extreme importance to the fact that education is a very time-intensive activity. Total time endowment is normalised to unity for both agent types. The time of the parent that is not employed on the market, denoted by H , together with the time of the child that is not employed for educational purposes, denoted by h , is used to produce a non-marketable household public good y , non-rivalrous and non-excludable within the family; for simplicity, no other input is required. A perfect substitute for the households public good, z , is available on the market at price p .

We assume that the parent is altruistically linked to the child. For simplicity, altruism is taken to be full, i.e. the parent weighs the kid's utility as her own. This is the simplest setting in which the model can be developed, and also one in which the cards are not too obviously stacked in favour of policy intervention, which indirectly reinforces our arguments. The arguments could however be adapted for a model based on the exchange view of the family.

The *laissez-faire* outcome

Let us begin by considering what would happen in a free market, in which there is no government intervention. The household production function (concave and increasing) is

$$y = y(H, h). \tag{1}$$

⁷The model could be recast in an OLG framework. All the results reached in the simpler case treated here would carry over, and we would have to add many unnecessary details, with the consequent risk of making the model lose its focus.

⁸Appendix A briefly discusses the case in which parents transfer to the children resources that can be directly consumed as well as resources meant to be invested in education.

We assume that the child cannot produce without a contribution from her parent, who must at least coordinate and supervise home production.⁹ Formally, H is an essential input, so that

$$y(0, h) = 0; \quad y_{Hh} = y_{hH} > 0. \quad (2)$$

Also, we introduce

$$x = x(e, d), \quad (3)$$

a strictly concave and increasing function representing the value of education for the child in consumption terms (the child's gross income). We assume that e and d are technological complements (the more time you spend on education, the more effective is the money you spend on it and *viceversa*), and that both time and money are essential to production,

$$x(0, d) = x(e, 0) = 0; \quad x_{ed} = x_{de} > 0. \quad (4)$$

Assuming additive separability for the utility functions, we write the parent's preferences as

$$[U(C) + F(y(H, h) + z)] + [u(x(e, d)) + f(y(H, h) + z)]. \quad (5)$$

We take the sub-utility functions $U(\cdot)$ and $u(\cdot)$ to be concave, whereas for $F(\cdot)$ and $f(\cdot)$ we require strict concavity; $U(\cdot)$ and $u(\cdot)$ refer to private consumption for the parent and the kid, respectively, and $F(\cdot)$ and $f(\cdot)$ refer to the household public good still for the parent and the kid, respectively.

Normalising the price of the consumption good to unity, the budget constraint for the parents is

$$C + pz + e = wl. \quad (6)$$

The time constraints for the parent and the kid, respectively, are

$$H + l = 1 \text{ and } h + d = 1. \quad (7)$$

Using these elements, we write the parent's problem as one of choosing C , H , z , e and h so as to

$$\begin{aligned} \text{Max} \quad & [U(C) + F(y(H, h) + z)] + [u(x(e, 1 - h)) + f(y(H, h) + z)] \\ \text{s.t.} \quad & C + pz + e - w(1 - H) = 0; \\ & H \geq 0; \quad z \geq 0; \quad h \geq 0. \end{aligned}$$

⁹This seems a reasonable assumption. Our qualitative results would carry over also in its absence but the exposition would be less straightforward.

For simplicity, and without any consequence for our policy analysis, we exclude from our consideration the equilibria where $H = 1$ and $C = 0$, implying that no time is devoted to market labour, or where $x = 0$, implying that no time and money is devoted to education.¹⁰ Moreover, we take it that there always is a positive level of consumption of the household public good (either y or z must exceed zero).

Given the above, the first order conditions (FOCs) are as follows:

$$U' = \lambda \quad (C); \quad (8)$$

$$(F' + f') y_H \leq \lambda w, \text{ plus complementary slackness } (H); \quad (9)$$

$$F' + f' \leq \lambda p, \text{ plus complementary slackness } (z); \quad (10)$$

$$u' x_e = \lambda \quad (e); \quad (11)$$

$$(F' + f') y_h \leq u' x_d, \text{ plus complementary slackness } (h); \quad (12)$$

where the subscripts denote partial derivatives, and λ is the marginal utility of income.

To begin with, let us then investigate the question whether the household public good is purchased on the market or produced internally. The intuition behind the analysis is straightforward: y and z are perfectly substitutable, therefore the parent will use the least expensive, or, to put it in a different language, she will act according to where her comparative advantage lies, in home production or in market work. The actual analysis is however somewhat complicated by the fact that the "price" of y is non-linear.

If the parent chooses home-production, and assuming an interior solution in which both

¹⁰If we were to consider an equilibrium with $x = 0$, it could be proved that the households where $d = e = 0$ would be either all those that produce the public good at home or the ones with a relatively higher wage among them. The existence of this equilibrium would not affect our argument.

time-inputs, H and h , are used,¹¹ it must be the case that

$$F' + f' = \frac{U'}{y_H}w \text{ and } F' + f' = \frac{u'}{y_h}x_d, \quad (13)$$

where we used (8), (9) and (12). In turn, this implies that the input mix in home-production is defined according to the following arbitrage equation:

$$U' \frac{w}{y_H} = u' \frac{x_d}{y_h}, \quad (14)$$

that is, the opportunity cost of the parent's home-time equals that of the child's home-time.

If the parent instead chooses to purchase z , (10) would become, using again (8):

$$F' + f' = U'p. \quad (15)$$

Then, comparing the rewritten FOCs (13) and (15), we can see that the household public good will be produced at home if $w/y_H < p$ (i.e. if the opportunity cost of home-production, measured by the ratio of the productivities on the market and at home, is less than the market price of the substitute) and purchased on the market otherwise.

It is important to establish how w/y_H varies with w . We assume the following:

$$\frac{\partial (w/y_H)}{\partial w} > 0, \quad (16)$$

which corresponds to the idea that y_H , at most, grows moderately (less than proportionally) as w increases.¹² As w and w/y_H increase, there will be some wage rate w^* for which

$$\frac{w^*}{y_H} = p. \quad (17)$$

¹¹Having already excluded equilibria where $z = 0$ and $h = 1$ ($d = 0$), this means that we do not consider the case where (9) is binding and (12) is slack, i.e., where $(F' + f')y_H = U'w$ and $(F' + f')y_h|_{h=0} < u'x_d|_{d=1}$, implying $H > 0$ and $h = 0$ ($d = 1$). Since we know from Appendix C that $dd/dw < 0$ (and $dh/dw > 0$) when $h > 0$, this could happen either for very low values of w , or for all the low wagers (i.e. all those households that produce y at home). In the former case our qualitative results would not be affected. In the latter case, instead, children are never employed in home production and thus equality of opportunity would be achieved. This however means that it is never profitable to employ children in home production, independently of the level of w . Since H is decreasing in w for the low wagers, as the comparative statics will tell us, it must be the case that $(F' + f')y_h|_{h=0} < u'x_d|_{d=1}$ holds even for those low wagers that have a relatively high w and thus a high level of $F' + f'$. This does not seem to represent a general case although it might, for example, represent an economy rapidly shifting from a rural to an industrial structure.

¹²For a similar assumption, see Becker and Murphy (2007).

Then, all households with $w < w^*$ produce y at home, while those with $w > w^*$ purchase z on the market.

Consider now the wage rate w^* : we can think of it as a threshold level at which the household is marginally indifferent between producing the public good at home and buying it externally and thus simultaneously produces y at home and buys z on the market.¹³ Indeed, at w^* , the FOCs (9), (10) and (12) all hold with equality, which implies that the mix in the production/consumption of the household public good is defined according to the following arbitrage equation:

$$U' \frac{w^*}{y_H} = u' \frac{x_d}{y_h} = U' p. \quad (18)$$

In general, we do not know whether w^* is unique, or rather there is an interval of wage rates for which (18) holds and households still simultaneously produce y at home and buy z on the market even if the proportion of y to z varies (as well as the levels of H and h). The policy analysis does not change much in either case: as we will see, our arguments are driven by the existence of a social divide between high-income parents who fully educate their children, and low-income parents who employ part of their kids' time for the domestic production process, and such a divide occurs independently of whether w^* is unique or not.

To keep things simple, we will therefore focus on the situations in which w^* is unique (Appendix B shows that, for example, this happens under the assumptions that $y_{hh} = y_{HH} = 0$ and $U'' = u'' = 0$). In such a case, we can establish the following threshold:

$$w^* = p y_H. \quad (19)$$

All households with $w < w^*$ produce y at home, while those with $w > w^*$ purchase z on the market; the intermediate group made only of the agents at $w = w^*$ consume a mix of home-produced and purchased public good.¹⁴

We will thus have:

1. a group of low-wage households, $w \in [0, w^*]$, where the kid's time is split between school and home-production ($h > 0$ and therefore $d < 1$), the parent works partly at home and

¹³This is due to the "price" of y being non linear and increasing, so that it is profitable to produce y only up to the point where its marginal price equals p .

¹⁴As H is decreasing in w for the households that produce y at home, we cannot exclude the case where $H = 0$ at w^* so that the public good will be completely purchased on the market. This however would not affect our analysis.

partly in the market ($H > 0$ and $l > 0$, with income wl), and the household public good is not purchased on the market ($z = 0$) except for those exactly at w^* (for whom $z > 0$);

2. a group of high-wage households, $w \in (w^*, \bar{w}]$, where the kids go to school full time ($h = 0$ and $d = 1$), the parents work full time ($H = 0$ and $l = 1$, with income w), and the household public good is purchased on the market ($z > 0$).

We can now describe the main comparative statics results.

Comparative statics

The details of the comparative statics analysis for the general case, where w^* is not necessarily unique, are reported in Appendix C: here, we give the main results and a few words of interpretation.

For the high wagers, we saw that $H = h = 0$, that is $l = d = 1$. Substituting the budget constraint (6) into the utility function (5), the optimisation problem becomes

$$\max_{z,e} [U(w - pz - e) + F(z)] + [u(x(e, 1)) + f(z)], \quad (20)$$

and the consumption mix is determined by the arbitrage condition

$$\frac{F' + f'}{p} = u'x_e. \quad (21)$$

The value for e that emerges is then combined with $d = 1$ to give the equilibrium value for x . The comparative statics results are:

$$\frac{\partial z}{\partial w} \geq 0; \quad \frac{\partial e}{\partial w} \geq 0. \quad (22)$$

Summing up, all the kids from the high-wage families are educated full time; expenditure on both the household public good and education are non-decreasing in the wage rate; as a consequence, so is x . Income is trivially increasing in the wage rate.

As for the low wagers, we know that $z = 0$ for $w < w^*$. So, the problem is

$$\max_{H,e,h} [U(w - e - wH) + F(y(H, h))] + [u(x(e, 1 - h)) + f(y(H, h))]. \quad (23)$$

The consumption mix is determined by

$$U' = u'x_e, \quad (24)$$

while the input mix (in the production of y) is determined by (18) above. The emerging values for e and d determine x . The comparative statics results are

$$\frac{\partial H}{\partial w} < 0; \quad \frac{\partial e}{\partial w} > 0; \quad \frac{\partial h}{\partial w} > 0. \quad (25)$$

The low wagers' labour supply is increasing in the wage rate, and therefore so is their income. Expenditure in education increases in the wage rate. However, the higher is w , the more the kids will be working at home: in other words, as w increases the low wagers replace their own home time with that of the kids as working outside home become more advantageous. We cannot say whether the total returns to education x increase, because less school-time makes expenditure less effective (d and e are complements).¹⁵

For the households at w^* , both $\frac{\partial H}{\partial w}$ and $\frac{\partial h}{\partial w}$ are obviously negative when w^* is unique but there are no unambiguous results available for the general case.

The characteristics of the market equilibrium

From the analysis above, it is clear that the economy presents an "educational divide". First of all, educational expenditure increases with the wage rate, so there is of course a source of difference there. But, mostly, the difference originates from the fact that those households whose wage is in the upper range give their children a full-time education, while the households with lower wage rates send their kids to school only for a part of their time, and employ the remaining time for the production of a household public good. Further, since time and expenditure are, plausibly, seen as complementary inputs in our model, it follows that the money that the high wagers spend is more effective than that spent by the low wagers, thereby further accentuating the divide.

The divide has nothing to do with altruism, which is just as strong for the low as for the high wagers. Indeed, the main force behind the separation between those who enjoy a full-time education and those who don't is the logic of *comparative advantages* for the production of the

¹⁵If one should make an assumption, it would probably be safe to postulate that increasing expenditure will entail little compensation for a reduced time at school, which is probably mostly irreplaceable due to the teacher's inputs, the peer effects, etc.. An alternative approach is that proposed by Glewwe (2002) for less developed countries where years of schooling and school quality are considered as alternative inputs in the production of the child cognitive skills. In that set-up, when the learning efficiency of the child increases or the cost of education decreases, parents prefer to increase the quality of the children education rather than the quantity because the latter has a greater opportunity cost in terms of less time for the child market work.

household public good. If such an advantage lies in home-production, i.e. w/y_H is lower than p , then the child will not achieve a full-time education.

If, as we argued in Section II above, it is plausible to assume that the government has (also) a horizontal equity objective such as granting equal opportunities to all the children, then, the educational divide is of course a matter of concern. In this case, it would clearly become desirable to devise some kind of policy package so as to allow also the low wagers to send their children to school full time.

IV Alternative policy frameworks

We now assess the performance of the policy packages that economists usually recommend. Since we are in an altruistic setting, we start with a linear income tax with tax rate $\tau > 0$ and lump-sum subsidy $T > 0$. In a beckerian model, using such a redistributive instrument would lead to a social equilibrium in which both vertical and horizontal equity objectives are attained (see Sections 1 and 2 above).

Our results, however, stand in stark contrast, as we show that *the income tax is not entirely effective as far as horizontal equity is concerned*. Things do not change if we add educational subsidies; in fact, we argue that no price instrument can guarantee equality of opportunity in our setting. In our framework, the presence of redistributive taxation, which is necessary to achieve some degree of vertical equity, damages horizontal equity. Therefore, we consider the merits of an alternative policy in which we replace the educational subsidies with a compulsory education package consisting in a given amount of per-child expenditure and a given number of years of mandatory schooling, and perform a full analysis of this case. Basically, this outcome is predicated on our characterisation of education as requiring both money *and time*: it is indeed the time allocation that cannot be controlled *via* the usual tax instruments.

Remarkably, the tax instruments remain ineffective, as far horizontal equity is concerned, also if upgraded to non-linearity. Not even personalised lump-sum taxes can remedy the lack of equal opportunities – intuitively this occurs because the marginal conditions ruling the allocation of time are not changed by lump-sum taxes (by the very definition of the latter). Also incentive-compatible non-linear policy instruments would still require a dedicated policy for horizontal equity purposes: actually, non-linear price controls are basically equivalent to personalised quantity controls in that they allow the social planner to control the agents’ time and budget allocations, so the requirement that all children go to school full-time in that setting is equivalent

to a compulsory education policy.

Hence, we have taken all tax instruments to be linear: this allows us to keep the model manageable as well as based on reasonable assumptions on what the policy makers can observe (non-linear taxes require knowledge of the level of consumption, purchase of the household public good, educational expenditure and hours of school for all households), without losing anything of substance in terms of results.

Redistributive taxation with educational subsidies

With an income tax in place (where τ is the marginal tax rate and T the uniform subsidy) plus educational subsidies for educational expenditure and hours of school (σ and s , respectively) the budget constraint becomes

$$C + pz + (1 - \sigma)e + (1 - \tau)wH = (1 - \tau)w + T + (1 - h)s. \quad (26)$$

We assume that also the future income of the children is taxed at the same rate as that of the parents so that children earn $(1 - \tau)x(\cdot)$ after tax. The utility function therefore is:

$$[U(C) + F(y(H, h) + z)] + [u((1 - \tau)x(e, 1 - h)) + f(y(H, h) + z)]. \quad (27)$$

The FOCs are as follows:

$$U' = \lambda \quad (C); \quad (28)$$

$$(F' + f')y_H \leq \lambda(1 - \tau)w, \text{ plus complementary slackness } (H); \quad (29)$$

$$F' + f' \leq \lambda p, \text{ plus complementary slackness } (z); \quad (30)$$

$$u'(1 - \tau)x_e = (1 - \sigma)\lambda \quad (e); \quad (31)$$

$$(F' + f')y_h \leq u'(1 - \tau)x_d + \lambda s \text{ plus complementary slackness } (h). \quad (32)$$

Note that

$$U' = u' \frac{(1 - \tau)x_e}{(1 - \sigma)} \quad (33)$$

by (28) and (31). If the parent chooses home-production, and assuming an interior solution in which both time-inputs, H and h , are used, it must be the case that

$$F' + f' = \frac{U'}{y_H}(1 - \tau)w \text{ and } F' + f' = \frac{u'}{y_h}(1 - \tau)x_d + \frac{u'}{y_h} \frac{s(1 - \tau)x_e}{1 - \sigma}, \quad (34)$$

where we used (28), (29), (32) and (33). The arbitrage condition for the allocation of time becomes:

$$\frac{U'}{y_H}w = \frac{u'}{y_h} \left(x_d + \frac{s}{1 - \sigma}x_e \right). \quad (35)$$

Notice that the subsidy s on school hours works as a way of increasing the return to expenditure in education. Money is fungible, and although in this case it is given for the time the kid spends at school it can be used for anything: the parent's altruism makes her split the amount of the subsidy between her consumption and the expenditure for the child, both of which are therefore actually subsidised.

If the parent instead chooses to purchase z , (30) would become, using again (28):

$$F' + f' = U'p. \quad (36)$$

Then, comparing the rewritten FOCs (34) and (36), we can see that the household public good will be produced at home if $(1 - \tau)w/y_H < p$ and purchased on the market otherwise. By replicating the analysis of the free-market case, we will find a wage rate

$$w^* = \frac{p}{1 - \tau} y_H \quad (37)$$

such that all households with $w < w^*$ produce y at home, those with $w > w^*$ purchase z on the market, and those at w^* do both. Suppose now that a social welfare maximisation exercise has been performed, and that the desired level of redistribution has been actuated via the optimally set tax instruments. In principle, the tax instruments may act on H (recall that the agents at w^* produce part of the household public good domestically) and thus on y_H , which would mean that w^* is a function of all of them. However, these would all be second-order effects. If we treat them as negligible, we are left with the main effect coming from τ :

$$\frac{\partial w^*}{\partial \tau} = \frac{p}{(1 - \tau)^2} y_H > 0. \quad (38)$$

Relative to the free-market situation, then, in an equilibrium with policy there will be *less* agents who educate their kids full time! This is because the marginal tax rate τ alters the comparative advantage situation: working outside home becomes less advantageous, and more agents choose to produce the household public good domestically. The achievement of a redistributive objective in vertical equity terms *via* the income tax would therefore limit equality of opportunity.¹⁶ The price subsidies are ineffective to remedy this. We would of course expect the subsidies to increase educational expenditure: but it is doubtful whether this might compensate

¹⁶Incidentally notice that also a tax on z , which we do not consider in the model to keep things simple, would have a negative effect on the threshold wage rate - much in the same way as an increase in the income tax rate. Since the tax would have mostly a redistributive purpose (hitting a good that is only consumed by the high-wagers) the logic is the same: vertical equity conflicts with horizontal equity.

the kids who have been obliged to leave full-time schooling for the reduced educational time (we have already made this point above – see fn. 15).¹⁷

In our setup, therefore, the standard beckerian prescription of using redistributive policy to make even the less well-off prone to educate their children is ineffective, despite the assumption of altruism within the family, as far as horizontal equity objectives are concerned. Rather, redistributive taxation has a negative effect, since the presence of a marginal tax rate forces some individuals not to educate their children full time. This cannot be remedied by the education subsidies, which mainly work on the intensive margin.

Redistributive taxation with a compulsory education package

Given the weak performance of the standard set of policy tools, it makes sense to investigate whether compulsory education can be more effective. Consider then, along with the linear income tax with tax rate $\tau > 0$ and lump-sum subsidy $\hat{T} > 0$, a compulsory education package (E, D) where E is per-child expenditure and D is the number of years of mandatory schooling. The parents can top up both rations, adding expenditure and school time beyond the mandatory level: then, e and d are now the amounts of expenditure and school-time that the parent can employ for topping up the compulsory levels, respectively.

We characterise the policy problem as one of choosing the policy tools that maximise a social welfare function, thereby satisfying a vertical equity requirement, subject to a constraint that imposes a horizontal equity requirement. The constraint is simply that D be set at unity, thus forcing $h = d = 0$: this way, all kids will go to school full time (setting a lower level would be self-defeating, as the high wagers would adjust in order to have $D + d = 1$, while the low wagers would continue to have $D + d < 1$). This corresponds to a current practice in virtually all Western countries today.¹⁸

¹⁷Part (or all) of the low wagers will see their income increase thanks to the poll subsidy. This will translate in a higher level of educational expenditure. Given the complementarity of e and d , this will make both d and x increase for those households who were below the threshold even without the policy. Those households that switch from high to low wagers however do not necessarily see their income increase (they could be net contributors to the tax system). In any case they will face a strong reduction in d (recall that there is a discontinuity in d at w^*) which is likely to imply a reduction in x even if their income and consequently e were to increase.

¹⁸A parallel can be drawn with health economics, where some theories of justice analyse the interpersonal distribution of health by establishing different types of condition upon outcomes that need to be satisfied before solving any social planner maximisation problem. For example, a common condition is that a minimum decent level of health has to be fulfilled for some specified groups, and no trade-off is allowed between such a goal and

We further investigate whether it is optimal to set also a fixed level of expenditure E along with the tax instruments; this means in principle that the total expenditure $E + e$ might differ for kids from different families, although we will focus below on a case in which it is the same for all.

By substituting the constraint that $D = 1$ into the utility function, the latter becomes

$$[U(C) + F(y(H, 0) + z)] + [u((1 - \tau)x(E + e, 1)) + f(y(H, 0) + z)], \quad (39)$$

while the budget constraint is

$$C + pz + e + (1 - \tau)wH = (1 - \tau)w + \hat{T}. \quad (40)$$

Notice that it is possible to write the budget constraint also as

$$C + pz + (e + E) + (1 - \tau)wH = (1 - \tau)w + T, \quad (41)$$

that is, as if the agent were paying the educational expenditure herself. In fact, as formally shown in Appendix D, T now also covers E .

The problem of the agent is then to maximise (39) by choice of C , H , z , and e s.t. (41) and the non-negativity constraints. The FOCs are as follows:

$$U' = \lambda \quad (C); \quad (42)$$

$$(F' + f')y_H \leq \lambda(1 - \tau)w, \text{ plus complementary slackness} \quad (H); \quad (43)$$

$$(F' + f') \leq \lambda p, \text{ plus complementary slackness} \quad (z); \quad (44)$$

$$u'(1 - \tau)x_e \leq \lambda \text{ plus complementary slackness} \quad (e). \quad (45)$$

The choice between home production and market purchase depends on the measure of comparative advantage, just as before. The threshold wage rate is as in (37), and the distinction between high- and low wagers works in the same way, with the added twist that, for the high wagers, the presence of $D = 1$ is of no consequence because this is what the parents would have chosen anyway.

The indirect utility for both types of parent can be written as a function of the policy instruments, $V = V(\tau, T, E)$, and the derivatives w.r.t. the policy tools are

$$\frac{\partial V}{\partial T} = \lambda > 0; \quad \frac{\partial V}{\partial \tau} = -\lambda w(1 - H) - u'x < 0; \quad (46)$$

$$\frac{\partial V}{\partial E} = u'(1 - \tau)x_e - \lambda < 0 \text{ if } e = 0. \quad (47)$$

any other (Williams and Cookson 2000).

Notice that $H > 0$ for the low wagers (including the agents at w^* – see Appendix E) and $H = 0$ for the high wagers in the expression for $\partial V/\partial \tau$. The sign of $\partial V/\partial E$ depends on whether E exceeds the quantity that the agent would have chosen in the free market or not: $\partial V/\partial E$ is negative if it does, equal to 0 otherwise.

The next step requires us to check the comparative statics in this new setting with the policy instruments. This task is made extremely cumbersome by the fact that there are many possibilities concerning the extent to which the compulsory educational package actually constrains family choices. We noticed that $D = 1$ is always infra-marginal for the high wagers, while for the low wagers the constraint will definitely bite. Instead E may or may not bite for both types. Here we focus on the case that seems more interesting,¹⁹ i.e. the one in which E constrains the choices of all households. Therefore, $e = 0$ for all agents. This is the scenario in which the quantity constraints interfere the most with the free choices of the agents: can in this extreme case those constraints be welfare-improving?

Let us start from the comparative statics (calculations are found in Appendix E). Notice that the high wagers only choose z . We find that

$$\partial z/\partial w \geq 0; \partial z/\partial \tau \leq 0; \partial z/\partial T \geq 0; \partial z/\partial E \leq 0. \quad (48)$$

As for the low wagers, they only choose H , and we find that

$$\partial H/\partial w < 0; \partial H/\partial \tau > 0; \partial H/\partial T \geq 0; \partial H/\partial E \leq 0. \quad (49)$$

Finally, the agents at w^* choose both H and z , and we find that

$$\begin{aligned} \partial H/\partial w < 0; \partial H/\partial \tau > 0; \partial H/\partial T = \partial H/\partial E = 0; \\ \partial z/\partial w > 0; \partial z/\partial \tau < 0; \partial z/\partial T \geq 0; \partial z/\partial E \leq 0. \end{aligned} \quad (50)$$

The interpretation is rather straightforward: it may be mentioned that the absence of any impact by either T or E on the time allocation for the agents at w^* is due to the fact that for them the shadow price of home time, $(1 - \tau)w^*$, equals the value of its marginal product, i.e. py_H – see (37).

¹⁹As far as the effects of the education policies are concerned, the conclusions in the other cases are analogous to those discussed here. The results for the other cases are available from the authors.

Optimal second-best policy

We now consider a second-best policy problem in which both vertical and horizontal equity requirements are accounted for. The first aspect is taken care of by the standard assumption that the government maximizes a quasi-concave social welfare function subject to the revenue constraint; the second aspect is instead represented by the presence of the compulsory education scheme that establishes $D = 1$.²⁰

Assuming that the objective function is a generalised utilitarian, and using the indirect utility functions defined in the previous sub-section, we may write

$$\begin{aligned} & \max_{\tau, T, E} \int_0^{\bar{w}} \beta(w) V(\tau, T, E; w) G(w) dw \\ \text{s.t. } & \tau \int_0^{\bar{w}} x(E, 1) G(w) dw + \tau \int_0^{\bar{w}} w G(w) dw - \tau \int_0^{w^*(\tau)} [w H(w) G(w)] dw - T = R, \end{aligned}$$

where $\beta(w)$, with $\beta' < 0$, represents a set of welfare weights, $G(w)$ is the wage distribution function, and R is a fixed revenue requirement. The FOCs are

$$\int_0^{\bar{w}} \beta(w) \frac{\partial V}{\partial \tau} G(w) dw + \mu \left\{ \int_0^{\bar{w}} x(E, 1) G(w) dw + \int_0^{\bar{w}} w G(w) dw - \int_0^{w^*(\tau)} w H(w) G(w) dw - \tau \int_0^{w^*(\tau)} w \frac{\partial H(w)}{\partial \tau} G(w) dw - \tau w^* H(w^*) G(w^*) \frac{\partial w^*}{\partial \tau} \right\} = 0; \quad (51)$$

$$\int_0^{\bar{w}} \beta(w) \frac{\partial V}{\partial T} G(w) dw - \mu \left\{ \tau \int_0^{w^*(\tau)} w \frac{\partial H(w)}{\partial T} G(w) dw + 1 \right\} = 0; \quad (52)$$

$$\int_0^{\bar{w}} \beta(w) \frac{\partial V}{\partial E} G(w) dw + \mu \tau \left\{ \int_0^{\bar{w}} x_e G(w) dw - \int_0^{w^*(\tau)} w \frac{\partial H(w)}{\partial E} G(w) dw \right\} = 0; \quad (53)$$

where the derivatives with respect to the indirect utility functions are given by (46) and (47) and μ is the multiplier of the government's budget constraint. We can then state the main results concerning the policy rules.

First, rearrange the second FOC:

$$\frac{\int_0^{\bar{w}} \beta(w) (\partial V / \partial T) G(w) dw}{\mu} - \tau \int_0^{w^*(\tau)} w \frac{\partial H(w)}{\partial T} G(w) dw = 1; \quad (54)$$

²⁰ Alternatively, the horizontal equity requirement could be accounted for by assuming a different social welfare function which could also capture a measure of inequality of educational opportunities as a negative externality for the society. For example, social welfare could be negatively affected by a higher variance in the distribution of x . In this case, the argument in favour of a compulsory education scheme reducing such a variance would find an even higher support. A different approach is followed by Gasparini and Pinto (2006) where the choice of the optimal policy in terms of cash versus in-kind transfers depends, among the others, on the degree of the rich people's concern about education quality dispersion. In this work, income redistribution serves to counter a negative externality coming from the fact that rich people's utility negatively depends on the difference between the average education quality levels of the rich and the poor groups in the society.

the net social marginal utility of income, inclusive of its effect on revenue and weighted by μ , equals unity. This is a standard result that characterises the optimal T .

The third FOC can be rearranged as follows:

$$\tau \left\{ \int_0^{\bar{w}} x_e G(w) dw - \int_0^{w^*(\tau)} w \frac{\partial H(w)}{\partial E} G(w) dw \right\} = - \frac{\int_0^{\bar{w}} \beta(w) (\partial V / \partial E) G(w) dw}{\mu}, \quad (55)$$

that is, at an interior solution, the marginal benefit of education expenditure in terms of increased revenue must equal the marginal cost in terms of forcing the agents out of the chosen consumption bundle – recall that $\partial V / \partial E < 0$ when the ration bites. Increased revenue depends on the fact that setting E higher leads to a larger future income of the children x , as well as to less home-production time, or equivalently more time devoted to market work ($\partial H / \partial E \leq 0$ when the ration bites), and therefore more taxable income from the parents.

This analysis presupposes that $\tau > 0$. In order to check whether this is the case, we can rearrange the first FOC. To this end, define

$$\Psi \equiv \int_0^{\bar{w}} x(E, 1) G(w) dw + \int_0^{\bar{w}} w G(w) dw - \int_0^{w^*(\tau)} w H(w) G(w) dw. \quad (56)$$

We can then write

$$\tau = \frac{\Psi + \int_0^{\bar{w}} \beta(w) (\partial V / \partial \tau) G(w) dw / \mu}{\int_0^{w^*(\tau)} w \frac{\partial H(w)}{\partial \tau} G(w) dw + w^* H(w^*) G(w^*) \frac{py_H}{(1-\tau)^2}}, \quad (57)$$

where we have used (38).

From the fact that $\partial H / \partial \tau > 0$ we deduce that the denominator in (57) is positive. This term represents the total revenue loss associated with a marginal increase in τ : the reduction in labour supply implies a reduction of tax base, and the fact that w^* varies in the same direction as τ implies a further reduction because more agents start employing home-production to get the household public good, and therefore work less. The larger is this term, the smaller will be τ .

The first term at the numerator, Ψ , is, as we just said, the marginal revenue gain from the tax. It is positive because

$$\int_0^{\bar{w}} x(E, 1) G(w) dw + \int_0^{\bar{w}} w G(w) dw > \int_0^{w^*(\tau)} w H(w) G(w) dw. \quad (58)$$

The second term at the numerator of (57) is negative because $\partial V / \partial \tau < 0$ and represents the marginal welfare loss. Therefore, if the revenue gain exceeds the welfare loss, the numerator is positive as well. In that case, we have $\tau > 0$; and the larger is the difference between the two terms above the line, the larger is the tax rate.

To sum up, in order to have $E > 0$ it must be the case that $\tau > 0$: only if some form of redistributive taxation is in place, it may be optimal to force (at least some of) the agents to spend on education more than they would have done in a free market. The desirability of the quantity controls is justified by the fact that they imply a gain in revenue terms. If this gain is large enough to compensate the costs in terms of displaced consumption, then some form of quantitative restriction is welfare-improving.²¹

A remarkable feature of the result is that redistributive taxation and education policy in the form of compulsory education appear to be strongly intertwined. Neither works without the other, if we assume that both horizontal and vertical equity are policy concerns. Redistributive taxation satisfies the vertical equity requirements but damages the future earnings of (some of) the children,²² as we know from the analysis of the impact of the tax rate on the threshold wage rate; horizontal equity thus requires a specific education policy, in the form of mandatory schooling. At the same time, the full package of compulsory education, in which agents are forced to provide resources to finance the expenditure on education for their children, cannot be optimal without redistributive taxation.

Finally, let us notice that, as far as educational expenditure is concerned, the logic of our result partially overlaps the one prevailing in the standard analyses of in-kind transfers (Balestrino 1999, 2000), in which the desirability of the quantity restriction depends upon its capability to increase revenue. In those analyses revenue is commonly generated through a substitution process in the consumption basket: the typical policy recommendation is that quantity constraints should be applied to commodities which are complementary to heavily taxed ones. Education, however, when publicly provided above the level freely chosen by the households generates revenue not through a chain of substitution effects, but by directly raising taxable income. Of course, this comment only concerns educational *expenditure* and relates to vertical equity issues. In our setting, however, compulsory schooling is recommended primarily on hor-

²¹It may be worth noting that, had we chosen D optimally rather than have it fixed at the outset, we would have reached a similar conclusion to that for E , that is, we would have found that it is socially desirable, on vertical equity grounds, to establish mandatory schooling. Of course, there would not have been, at this level of generality, any guarantee that the optimal level of D were exactly unity, so in this sense we would not have necessarily achieved a full equality of opportunity.

²²As we have discussed in footnote 17, the redistributive policy will benefit the children of those households that would be low wagers even without the policy but would damage the children of the households that switch from high to low wagers.

horizontal equity grounds: no price instrument is capable to correct the behaviour of those agents who prefer to employ their children's *time* for domestic production rather than for education. From this perspective, the logic underpinning our result differs from the one employed in the standard analyses of in-kind transfers.

Our result also differs from Cigno (2012) that prescribes compulsory schooling as part of the optimal policy in a developing country setting where children spend part of their time in performing covert labour (equivalent to our home production) and may devote some other time to working on the market. The rest of the time is spent at school but "effective" education time also depends on the other activities (a tired kid or a kid who has no time for homework has a low effective education time). Compulsory enrolment (not a precise school time as in our model) becomes optimal due to asymmetric information on the use of the children's time, but is not needed in the absence of informational problems when lump-sum taxes and subsidies allow to achieve both efficiency and perfect equity.

V Concluding remarks

We began by asking whether there is a reason why education policy should involve a mandatory and (virtually) free-of-charge schooling period, as it commonly does in the Western countries. Economists should be particularly interested in obtaining an answer, as quantity controls are traditionally considered outperformed by price controls in standard economic theory. Having established that education policy must, for some reason, be implemented, many would argue that it should take the form of a price subsidy (making education less costly should make agents more prone to purchase it for their children) or simply be embedded in tax policy (redistributing resources in favour of the poor should automatically help them to send their children to school).

Now, it is well-known, at least since Guesnerie and Roberts (1984), that the superiority of price controls is only valid in first-best, and that quantity controls can be welfare-improving in a variety of second-best contexts: the last 20 years have seen a vast research effort on this that traces its roots to the contributions of Blomquist and Christiansen (1995) and Boadway and Marchand (1995) and continues to this day (e.g. Blomquist et al. 2010). Our work follows this stream of the literature, and aims to fill a blank space, because none of those works has dealt specifically with education as we believe it should be characterised, namely as i) an extremely expensive and time-consuming process that ii) involves a decision-maker (the parent) who is not the direct beneficiary (the child).

Using a model that accounts for both these features, we have first depicted a free market situation in which some agents ("high wagers") educate their children full time and spend a sizable amount of resources on them, while others ("low wagers") educate them only partially (and in principle might even not educate them at all).²³ This outcome is generated by the presence of an alternative usage of the children's time: rather than be sent to school, they can be employed in producing a household public good. The high wagers can afford to replace this home-produced good with a marketed substitute; the low wagers' comparative advantage, instead, lies in home-production. The free-market equilibrium is iniquitous where parents are concerned, due to their having different exogenous skills and thus different incomes (a vertical equity problem), and also where children are concerned, even if the parents are fully altruistic, because the kids receive different educations depending on whether they are born in a high-wage or a low-wage family. Further, the differences in the education they receive today imply that there will be a disparity in earning abilities tomorrow due to choices made by the parents, not by themselves (a horizontal equity problem).²⁴

Public policy is thus called for, both for vertical and horizontal equity reasons. In this framework, we argued that it is indeed socially optimal to introduce a compulsory education package, using a standard redistributive tax system to finance it. Mandatory schooling fully compensates the kids for the disadvantages at which their parents' choices might have put them. Adding a mandatory expenditure requirement forces the parents away from their equilibrium choices, which is of course costly but entails also an advantage in terms of increased revenue that can be used for redistributive purposes. Indeed, it may finance the poll subsidy, that goes to the family as a whole, and the educational expenditure for the children, including those from the less well-off families.

From the point of view of horizontal equity, a compulsory education policy is shown to

²³This result can be linked to the recent empirical literature that tries to analyse how exogenous changes in parents' education due to variations in compulsory schooling laws may affect the intergenerational transmission of education. For example, Piopiunik (2014) provides evidence that individuals with more schooling (and thus on average higher wages) value their kids' education more highly.

²⁴Another possible reason for public intervention, which we do not explore but just mention briefly here, is the fact that the children, despite all having the same ability, are educated at different level: this might indeed have efficiency implications. The *laissez-faire* equilibrium is clearly efficient from the point of view of the parents (or of the families as a whole), but if we look at it from the point of view of the children (e.g. if the social welfare function were given by the sum of children's sub-utility functions) this is no longer the case.

be superior to the use of price subsidies, that only work on the intensive margin, i.e. boost education expenditure for those who would have educated their children full time anyway in a free market, but are unable to induce those who didn't educate their kids full time to start doing so. And we also argued that redistributive taxation alone is in fact counter-productive, as it forces more agents than in *laissez-faire* to avoid educating their children full time, because it tips the comparative advantage balance in favour of making child household work more desirable. This suggests that redistributive taxation and compulsory education are best seen as complementary policies if we assume that the government pursues both vertical and horizontal equity objectives.

Appendix A - A model of consumer choice with bequests

In the main text we assume that parents can transfer resources to their children only through investments in education. Here, we allow the parents to transfer resources that can be directly used for consumption, such as, for example, bequests. Let then the parent choose the transfer c . Parent's preferences are

$$[U(C) + F(y(H, h) + z)] + [u(c + x(e, d)) + f(y(H, h) + z)], \quad (\text{A1})$$

and the budget constraint is

$$C + pz + c + e = wl. \quad (\text{A2})$$

The time constraints for the parent and the kid, respectively, are

$$H + l = 1 \text{ and } h + d = 1. \quad (\text{A3})$$

Using these elements, we write the parent's problem as one of choosing C , H , z , c , e and h so as to

$$\begin{aligned} \text{Max } & [U(C) + F(y(H, h) + z)] + [u(x(e, 1 - h)) + f(y(H, h) + z)] \\ \text{s.t. } & C + pz + e + c - w(1 - H) = 0; \\ & H \geq 0; \ z \geq 0; \ c \geq 0; \ e \geq 0; \ h \geq 0. \end{aligned}$$

The first order conditions (FOCs) are as follows:

$$U' = \lambda \quad (C); \quad (A4)$$

$$(F' + f') y_H \leq \lambda w, \text{ plus complementary slackness } (H); \quad (A5)$$

$$F' + f' \leq \lambda p, \text{ plus complementary slackness } (z); \quad (A6)$$

$$u' = \lambda \quad (c); \quad (A7)$$

$$u' x_e = \lambda \quad (e); \quad (A8)$$

$$(F' + f') y_h \leq u' x_d \text{ plus complementary slackness } (h). \quad (A9)$$

Our first question is whether the parent prefers to transfer resources to the child using the consumption good or investing in education. From (A7) and (A8), it is immediate to see that

$$(x_e > 1) \longrightarrow (u' < \lambda; u' x_e = \lambda) \longrightarrow (c = 0, e > 0); \quad (A10)$$

$$(x_e < 1) \longrightarrow (u' = \lambda; u' x_e < \lambda) \longrightarrow (c > 0, e = 0), \quad (A11)$$

while $x_e = 1$ would lead to an undetermined result. The outcome is very sharp because c and x are (reasonably) perfect substitutes. The intuition is clear: each unit of consumption that the parent transfers forward becomes, within the assumptions of the present model, exactly one unit of extra consumption for the child, while each unit of consumption that the parent transforms into one unit of educational expenditure may become more or less than one unit of extra consumption for the child depending on the returns to said expenditure. The result can clearly be generalised to more complicated settings: the parent will always choose the most efficient way of transferring resources to the child.

The model that we consider in the paper is basically one in which $x_e > 1$ everywhere for all households. In the opposite extreme case in which $x_e < 1$ everywhere for all households, we would have $e = 0$, which implies $d = 0$ because both inputs are essential to the production of x ; we would not have any education at all, which would make the model uninteresting for our purposes.

Mixed situations could be discussed, though. If some agents choose to educate their children while others rely on consumption transfers, the horizontal equity problem would be exacerbated. An especially clear case would emerge if, for example, the marginal returns to education x_e were to depend on the parent's wage: we might assume, in line with the empirical literature (e.g. Mayer 1997 and Blau 1999) that the parent's marketable skills (her wage in our model) and the child's ability to learn and profit from what she has learned (her $x(\cdot)$ function in our model)

are positively, although not necessarily perfectly, correlated. If so, it might well be the case that most of the high wagers in our model send their kids to school while most of the low wagers do not – thereby worsening the horizontal equity issues.

Appendix B - An example in which w^* is unique

From (5) and (6), the parent's maximization problem is

$$\max_{H,e,d,z} U(w - e - pz - wH) + F(y(H, 1 - d) + z) + u(x(e, d)) + f(y(H, 1 - d) + z), \quad (\text{B1})$$

where U , u and y are concave, F , f and x are strictly concave. We consider the special case where i) $y_{HH} = y_{hh} = 0$, and ii) $U'' = u'' = 0$; also, recall that $x_{ed} > 0$ and $y_{hH} > 0$ by assumption. In order to prove that w^* corresponds to a point and not to an interval, we must prove that the FOCs can simultaneously hold as equalities for only one value of w . Focusing on an interior solution for all of the four variables H , e , d , z , the FOCs are

$$-U'w + (F' + f')y_H = 0 \quad (H); \quad (\text{B2})$$

$$-U' + u'x_e = 0 \quad (e); \quad (\text{B3})$$

$$-(F' + f')y_h + u'x_d = 0 \quad (d); \quad (\text{B4})$$

$$-U'p + F' + f' = 0 \quad (z). \quad (\text{B5})$$

By totally differentiating, we have:

$$\begin{aligned} & [-U']dw + [(F'' + f'')(y_H)^2]dH + [0]de + \\ & - [(F'' + f'')y_H y_h + (F' + f')y_{Hh}]dd + [(F'' + f'')y_H]dz = 0; \end{aligned} \quad (\text{B6})$$

$$- [0]dw + [0]dH + [u'x_{ee}]de + [u'x_{ed}]dd + [0]dz = 0; \quad (\text{B7})$$

$$\begin{aligned} & [0]dw - [(F'' + f'')y_h y_H + (F' + f')y_{hH}]dH + [u'x_{de}]de + \\ & + [(F'' + f'')(y_h)^2 + u'x_{dd}]dd - [(F'' + f'')y_h]dz = 0; \end{aligned} \quad (\text{B8})$$

$$[0]dw + [(F'' + f'')y_H]dH + [0]de - [(F'' + f'')y_h]dd + [F'' + f'']dz = 0. \quad (\text{B9})$$

Now, let

$$\begin{bmatrix} a_{11} \\ a_{21} \\ a_{31} \\ a_{41} \end{bmatrix} = \begin{bmatrix} (F'' + f'')(y_H)^2 < 0 \\ 0 \\ -[(F'' + f'')y_H y_H + (F' + f')y_{Hh}] \\ (F'' + f'')y_H < 0 \end{bmatrix}, \quad \begin{bmatrix} a_{12} \\ a_{22} \\ a_{32} \\ a_{42} \end{bmatrix} = \begin{bmatrix} 0 \\ u'x_{ee} < 0 \\ u'x_{ed} > 0 \\ 0 \end{bmatrix},$$

$$\begin{bmatrix} a_{13} \\ a_{23} \\ a_{33} \\ a_{43} \end{bmatrix} = \begin{bmatrix} -[(F'' + f'')y_H y_h + (F' + f')y_{Hh}] \\ u'x_{de} > 0 \\ (F'' + f'')(y_h)^2 + u'x_{dd} < 0 \\ -(F'' + f'')y_h > 0 \end{bmatrix}, \quad \begin{bmatrix} a_{14} \\ a_{24} \\ a_{34} \\ a_{44} \end{bmatrix} = \begin{bmatrix} (F'' + f'')y_H < 0 \\ 0 \\ -(F'' + f'')y_h > 0 \\ F'' + f'' < 0 \end{bmatrix},$$

where the sign of $a_{13} = a_{31}$ is not determined. Then

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ a_{41} & a_{42} & a_{43} & a_{44} \end{bmatrix} \begin{bmatrix} dH/dw \\ de/dw \\ dd/dw \\ dz/dw \end{bmatrix} = \begin{bmatrix} U' \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad (\text{B10})$$

and the comparative statics signs are as follows. Take dH/dw first:

$$\text{sgn } dH/dw = \text{sgn } U' \begin{vmatrix} a_{22} & a_{23} & a_{24} \\ a_{32} & a_{33} & a_{34} \\ a_{42} & a_{43} & a_{44} \end{vmatrix} < 0, \quad (\text{B11})$$

as the determinant is negative because of the maximization condition. Consider then de/dw :

$$\text{sgn } de/dw = -\text{sgn } U' \begin{vmatrix} a_{21} & a_{23} & a_{24} \\ a_{31} & a_{33} & a_{34} \\ a_{41} & a_{43} & a_{44} \end{vmatrix}. \quad (\text{B12})$$

Given that $a_{21} = a_{24} = 0$, it is

$$\begin{aligned} \text{sgn } de/dw &= -\text{sgn } U'(-a_{23})(a_{31}a_{44} - a_{41}a_{34}) = \\ &= \text{sgn } U'u'x_{de} [-(F'' + f'')(F' + f')y_{Hh}] > 0. \end{aligned} \quad (\text{B13})$$

Consider then dd/dw :

$$\text{sgn } dd/dw = \text{sgn } U' \begin{vmatrix} a_{21} & a_{22} & a_{24} \\ a_{31} & a_{32} & a_{34} \\ a_{41} & a_{42} & a_{44} \end{vmatrix}. \quad (\text{B14})$$

Given that $a_{21} = a_{24} = 0$, and that $a_{31}a_{44} - a_{41}a_{34} = -(F'' + f'')(F' + f')(y_{Hh}) > 0$, it is

$$\text{sgn } dd/dw = \text{sgn } U' a_{22}(a_{34}a_{41} - a_{31}a_{44}) = u' x_{ee}(F'' + f'')(F' + f')(y_{Hh}) > 0. \quad (\text{B15})$$

Consider finally dz/dw :

$$\text{sgn } dz/dw = -\text{sgn } U' \begin{vmatrix} a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \\ a_{41} & a_{42} & a_{43} \end{vmatrix}. \quad (\text{B16})$$

Given that $a_{21} = a_{42} = 0$, it is

$$\begin{aligned} \text{sgn } dz/dw &= -\text{sgn } \{U' a_{41}(a_{22}a_{33} - a_{32}a_{23}) - a_{31}a_{22}a_{43}\} = \\ &= -\text{sgn } U' \left\{ (F'' + f'') y_H \left[u' x_{ee} \left((F'' + f'')(y_h)^2 + u' x_{dd} \right) - (u' x_{ed})^2 \right] + \right. \\ &\quad \left. - [-(F'' + f'') y_h y_H + (F' + f') y_{Hh}] u' x_{ee} [-(F'' + f'') y_h] \right\} = \\ &= -\text{sgn } U' \left\{ (F'' + f'') \left[y_H x_{dd} - y_H (u' x_{ed})^2 + (F' + f') y_{Hh} (u')^2 x_{ee} y_h \right] \right\} < 0 \end{aligned} \quad (\text{B17})$$

We then have:

$$dH/dw < 0, \quad de/dw > 0, \quad dd/dw > 0, \quad dz/dw < 0. \quad (\text{B18})$$

We can use this comparative statics to prove that it is not possible to have the four FOCs holding as equalities over an interval of values of w . In fact, for the four FOCs to hold as equalities it must be the case that

$$\frac{w}{y_H} = p, \quad (\text{B19})$$

over the whole interval but this would imply that h has to increase as w increases, contradicting $dd/dw > 0$. Hence, with this specification, w^* is unique.

Appendix C - Comparative statics in *laissez-faire*

Recall that we are using a separable utility function throughout, and that we require $U(\cdot)$ and $u(\cdot)$ to be concave, $F(\cdot)$, $f(\cdot)$ and $x(\cdot)$ to be strictly concave.

High wagers ($w > w^*$)

In the case of high wagers, we can write the maximisation problem as

$$\max_{z, e} (U(w - pz - e) + F(z)) + (u(x(e, 1)) + f(z)). \quad (\text{C1})$$

The FOCs are

$$-U'p + F' + f' = 0 \quad (z); \quad (C2)$$

$$-U' + u'x_e = 0 \quad (e). \quad (C3)$$

By totally differentiating, we have:

$$-U''p dw + [U''p^2 + F'' + f''] dz + U''p de = 0; \quad (C4)$$

$$-U''dw + U''p dz + \left[U'' + \left(u''(x_e)^2 + u'x_{ee} \right) \right] de = 0. \quad (C5)$$

Therefore:

$$\begin{bmatrix} U''p^2 + F'' + f'' & U''p \\ U''p & U'' + (u''(x_e)^2 + u'x_{ee}) \end{bmatrix} \begin{bmatrix} dz/dw \\ de/dw \end{bmatrix} = \begin{bmatrix} U''p \\ U'' \end{bmatrix}. \quad (C6)$$

Then, the signs are as follows

$$\text{sgn } dz/dw = \text{sgn } \left[U''p \left(U'' + (u''(x_e)^2 + u'x_{ee}) \right) \right] - (U'')^2 p \geq 0; \quad (C7)$$

$$\text{sgn } de/dw = \text{sgn } \left[(U''p^2 + F'' + f'') U'' - (U''p)^2 \right] \geq 0. \quad (C8)$$

Low wagers ($w < w^*$)

In the case of low wagers, we can write the maximisation problem as

$$\max_{H,e,d} U(w - e - wH) + F(y(H, 1 - d)) + u(x(e, d)) + f(y(H, 1 - d)). \quad (C9)$$

The FOCs for an interior solution are:

$$-U'w + (F' + f') y_H = 0 \quad (H); \quad (C10)$$

$$-U' + u'x_e = 0 \quad (e); \quad (C11)$$

$$-(F' + f') y_h + u'x_d = 0 \quad (d). \quad (C12)$$

Totally differentiating, we have:

$$\begin{aligned} & -[U''(1 - H)w + U'] dw + [U''w^2 + (F'' + f'')(y_H)^2 + (F' + f') y_{HH}] dH + [U''w] de - \\ & -[(F'' + f'') y_H y_h + (F' + f') y_{Hh}] dd = 0; \end{aligned} \quad (C13)$$

$$\begin{aligned} & -[U''(1 - H)] dw + [U''w] dH + \left[U'' + (u''(x_e)^2 + u'x_{ee}) \right] de + \\ & + [u''x_e x_d + u'x_{ed}] dd = 0; \end{aligned} \quad (C14)$$

$$\begin{aligned} & 0dw - [(F'' + f'') y_h y_H + (F' + f') y_{hH}] dH + [u''x_d x_e + u'x_{de}] de + \\ & + [(F'' + f'')(y_h)^2 + (F' + f') y_{hh} + (u''(x_d)^2 + u'x_{dd})] dd = 0. \end{aligned} \quad (C15)$$

Now, let

$$\begin{bmatrix} a_{11} \\ a_{21} \\ a_{31} \end{bmatrix} = \begin{bmatrix} U''w^2 + (F'' + f'')(y_H)^2 + (F' + f')y_{HH} < 0 \\ U''w \leq 0 \\ -[(F'' + f'')y_h y_H + (F' + f')y_{hH}] \end{bmatrix}; \quad (\text{C16})$$

$$\begin{bmatrix} a_{12} \\ a_{22} \\ a_{32} \end{bmatrix} = \begin{bmatrix} U''w \leq 0 \\ U'' + u''(x_e)^2 + u'x_{ee} < 0 \\ u''x_e x_d + u'x_{ed} \end{bmatrix}; \quad (\text{C17})$$

$$\begin{bmatrix} a_{13} \\ a_{23} \\ a_{33} \end{bmatrix} = \begin{bmatrix} -[(F'' + f'')y_h y_h + (F' + f')y_{Hh}] \\ u''x_d x_e + u'x_{de} \\ (F'' + f'')(y_h)^2 + (F' + f')y_{hh} + u''(x_d)^2 + u'x_{dd} < 0 \end{bmatrix}, \quad (\text{C18})$$

where the signs of a_{31} , a_{13} , a_{32} and a_{23} are not determined. Then

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \begin{bmatrix} dH/dw \\ de/dw \\ dd/dw \end{bmatrix} = \begin{bmatrix} [U''(1-H)w + U'] \\ U''(1-H) \\ 0 \end{bmatrix}. \quad (\text{C19})$$

Further assuming that

$$-\frac{U''}{U'} < \frac{1}{(1-H)w}, \quad (\text{C20})$$

the comparative statics signs are as follows. Take dH/dw first:

$$\begin{aligned} \text{sgn } dH/dw &= -\text{sgn} \left\{ \underbrace{\left[U''(1-H)w + U' \right]}_{+} \left(\underbrace{a_{22}a_{33} - a_{32}a_{23}}_{+} \right) \right. \\ &\quad \left. + \underbrace{U''(1-H)}_{-} \left(\underbrace{a_{23}a_{31} - a_{21}a_{33}}_{?} \right) \right\}. \end{aligned} \quad (\text{C21})$$

Since $(a_{22}a_{33} - a_{32}a_{23})$ is positive because of the maximization condition, the first term is also positive. For $U'' < 0$, the sign of the second term is ambiguous. To sign it, we assume that there is a moderate complementarity both between d and e and between h and H , namely we suppose that

$$\begin{aligned} -\frac{u''}{u'} &> \frac{x_{de}}{x_d x_e}, \\ -\frac{F'' + g''}{F' + g'} &> \frac{y_{hH}}{y_h y_H}, \end{aligned}$$

which imply that $a_{23} < 0$ and $a_{31} > 0$, respectively. Since $(a_{23}a_{31} - a_{21}a_{33})$ is then non positive, it follows that

$$dH/dw < 0 \text{ and } dl/dw > 0. \quad (\text{C22})$$

Consider then de/dw :

$$\begin{aligned} & \text{sgn } de/dw = \\ & -\text{sgn} \left\{ \underbrace{U''(1-H)}_{-} \underbrace{(a_{11}a_{33} - a_{31}a_{13})}_{+} + \underbrace{[U''(1-H) + U']}_{+} \underbrace{\begin{pmatrix} a_{32}a_{13} & -a_{12} & a_{33} \\ ? & - & - \end{pmatrix}}_{?} \right\} \end{aligned} \quad (\text{C23})$$

We know that the term $a_{11}a_{33} - a_{31}a_{13}$ is positive because of the maximization conditions, and, because of our previous assumptions, we are considering the case where $a_{32}a_{13} - a_{12}a_{33} \leq 0$; then,

$$de/dw > 0. \quad (\text{C24})$$

Finally, let us consider dd/dw :

$$\begin{aligned} & \text{sgn } dd/dw = \\ & = -\text{sgn} \left\{ \underbrace{[U''(1-H) + U']}_{+} \underbrace{\begin{pmatrix} a_{12}a_{23} & -a_{13}a_{22} \\ - & ? & - \end{pmatrix}}_{?} + \underbrace{U''(1-H)}_{-} \underbrace{\begin{pmatrix} a_{21}a_{13} & -a_{11} & a_{23} \\ - & ? & - \end{pmatrix}}_{?} \right\} \end{aligned} \quad (\text{C25})$$

Since $a_{23} < 0$ and $a_{31} > 0$, it immediately follows that

$$dd/dw < 0 \text{ and } dh/dw > 0. \quad (\text{C26})$$

Appendix D - Equivalence of revenue constraint

To see that public budget constraint (40) is equivalent to budget constraint (41), let us first write the government budget if E is paid for by the government itself and then if E is paid by the parent:

$$\tau \int_0^{\bar{w}} w(1-H(w))G(w)dw + \tau \int_0^{\bar{w}} x(E+e, D+d)G(w)dw - E = \widehat{T}, \quad (\text{D1})$$

$$\tau \int_0^{\bar{w}} w(1-H(w))G(w)dw + \tau \int_0^{\bar{w}} x(E+e, D+d)G(w)dw = T. \quad (\text{D2})$$

To check the equivalence, integrate (40) to yield

$$\int_0^{\bar{w}} (C + pz + e) G(w) dw - (1 - \tau) \int_0^{\bar{w}} w (1 - H) G(w) dw = \hat{T}, \quad (\text{D3})$$

and substitute the revenue constraint (D1); then integrate (41) to yield

$$\int_0^{\bar{w}} (C + pz + e) G(w) dw + E - (1 - \tau) \int_0^{\bar{w}} w (1 - H) G(w) dw = T, \quad (\text{D4})$$

and substitute (D2) (recall that the agents have unit mass). It is immediate to see that the resource constraints computed using the two procedures coincide:

$$\begin{aligned} \int (C + pz + e) G(w) dw + E + \tau \int_0^{\bar{w}} x (E + e, D + d) G(w) dw = \\ = \int_0^{\bar{w}} w (1 - H) G(w) dw. \end{aligned} \quad (\text{D5})$$

Appendix E - Comparative statics under compulsory education

High wagers ($w > w^*$)

Having no need to employ their time in home production, the high wagers set $H = h = 0$: all the parent's time goes into working and all the kid's time goes into education. The constraint that $D = 1$ is of no consequence because that is what the parents would have chosen anyway. On the contrary, E is an actual constraint ($e = 0$). Then, high wagers choose z to maximise

$$[U((1 - \tau)w + T - pz - E) + F(z)] + [u((1 - \tau)x(E, 1)) + f(z)]. \quad (\text{E1})$$

The FOC is:

$$-U'p + F' + f' = 0, \quad (\text{E2})$$

and it follows that

$$\frac{\partial z}{\partial w} = -\frac{-U''p(1 - \tau)}{U''p^2 + F'' + f''} \geq 0; \quad \frac{\partial z}{\partial \tau} = -\frac{U''wp}{U''p^2 + F'' + f''} \leq 0; \quad (\text{E3})$$

$$\frac{\partial z}{\partial T} = -\frac{-U''p}{U''p^2 + \alpha F'' + f''} \geq 0; \quad \frac{\partial z}{\partial E} = -\frac{U''p}{U''p^2 + F'' + f''} \leq 0. \quad (\text{E4})$$

Low wagers ($w < w^*$)

Low wagers are constrained by both E and D ($d = 0$, $e = 0$). Consequently, they only choose H to maximise

$$[U((1 - \tau)w + T - E - (1 - \tau)wH) + F(y(H, 0))] + [u((1 - \tau)x(E, 1)) + f(y(H, 0))]. \quad (\text{E5})$$

The FOC is

$$-(1-\tau)U'w + (F' + f')y_H = 0. \quad (\text{E6})$$

Hence, since

$$-\frac{U''}{U'} < \frac{1}{(1-\tau)w(1-H)}, \quad (\text{E7})$$

from (C20), we have that

$$\begin{aligned} \frac{\partial H}{\partial w} &= -\frac{-(1-\tau)U' - (1-\tau)^2 w(1-H)U''}{U''(1-\tau)^2 w^2 + (F' + f')y_{HH} + (F'' + f'')(y_H)^2} = \\ &\quad -\frac{-(1-\tau)(U' + (1-\tau)w(1-H)U'')}{U''(1-\tau)^2 w^2 + (F' + f')y_{HH} + (F'' + f'')(y_H)^2} < 0; \end{aligned} \quad (\text{E8})$$

$$\frac{\partial H}{\partial \tau} = -\frac{w[U' + (1-\tau)w(1-H)U'']}{U''(1-\tau)^2 w^2 + (F' + f')y_{HH} + (F'' + f'')(y_H)^2} > 0; \quad (\text{E9})$$

$$\frac{\partial H}{\partial T} = -\frac{-(1-\tau)wU''}{U''(1-\tau)^2 w^2 + (F' + f')y_{HH} + (F'' + f'')(y_H)^2} \geq 0; \quad (\text{E10})$$

$$\frac{\partial H}{\partial E} = -\frac{(1-\tau)wU''}{U''(1-\tau)^2 w^2 + (F' + f')y_{HH} + (F'' + f'')(y_H)^2} \leq 0. \quad (\text{E11})$$

w^* -wagers

The agents at w^* are constrained by both E and D ($d = 0$, $e = 0$); they choose H and z to maximise

$$\begin{aligned} &U((1-\tau)w^* + T - E - pz - (1-\tau)w^*H) + F(y(H, 0) + z) + \\ &\quad + u((1-\tau)x(E, 1)) + f(y(H, 0) + z). \end{aligned} \quad (\text{E12})$$

The FOCs are

$$-U'(1-\tau)w^* + (F' + f')y_H = 0 \quad (H); \quad (\text{E13})$$

$$-U'p + F' + f' = 0 \quad (z). \quad (\text{E14})$$

Hence

$$\frac{(1-\tau)w^*}{y_H} = p, \quad (\text{E15})$$

as expected. Total differentiation yields

$$\begin{aligned}
& - \left[(1-\tau)U' + (1-\tau)^2(1-H)w^*U'' \right] dw^* + \left[U'w^* + (w^*)^2(1-\tau)(1-H)U'' \right] d\tau + \\
& \quad - \left[(1-\tau)w^*U'' \right] dT + \left[(1-\tau)w^*U'' \right] dE + \\
& \quad \left[(1-\tau)^2(w^*)^2U'' + (F'' + f'')(y_H)^2 + (F' + f')(y_{HH}) \right] dH + \\
& \quad + \left[(1-\tau)w^*pU'' + (F'' + f'')y_H \right] dz = 0; \tag{E16}
\end{aligned}$$

$$\begin{aligned}
& - \left[(1-\tau)(1-H)pU'' \right] dw^* + \left[w^*(1-H)pU'' \right] d\tau - \left[U''p \right] dT + \left[U''p \right] dE + \\
& \quad + \left[(1-\tau)w^*pU'' + (F'' + f'')y_H \right] dH + \left[p^2U'' + F'' + f'' \right] dz = 0. \tag{E17}
\end{aligned}$$

Now, let

$$\begin{aligned}
\begin{bmatrix} a_{11} \\ a_{21} \end{bmatrix} &= \begin{bmatrix} \left((1-\tau)^2(w^*)^2 \right) U'' + (F'' + f'')(y_H)^2 + (F' + f')(y_{HH}) < 0 \\ (1-\tau)w^*pU'' + (F'' + f'')y_H < 0 \end{bmatrix}, \\
\begin{bmatrix} a_{12} \\ a_{22} \end{bmatrix} &= \begin{bmatrix} (1-\tau)w^*pU'' + (F'' + f'')y_H < 0 \\ p^2U'' + F'' + f'' < 0 \end{bmatrix}.
\end{aligned}$$

Then:

$$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} dH/dw \\ dz/dw \end{bmatrix} = \begin{bmatrix} (1-\tau)(U' + (1-\tau)(1-H)w^*U'') \\ (1-\tau)(1-H)U''p \end{bmatrix}; \tag{E18}$$

$$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} dH/d\tau \\ dz/d\tau \end{bmatrix} = \begin{bmatrix} -w^*(U' + w^*(1-\tau)(1-H)U'') \\ -w^*(1-H)U''p \end{bmatrix}; \tag{E19}$$

$$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} dH/dT \\ dz/dT \end{bmatrix} = \begin{bmatrix} (1-\tau)w^*U'' \\ U''p \end{bmatrix}; \tag{E20}$$

$$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} dH/dE \\ dz/dE \end{bmatrix} = \begin{bmatrix} -(1-\tau)w^*U'' \\ -U''p \end{bmatrix}. \tag{E21}$$

Using (C20) and the fact that $(1-\tau)w^* = py_H$ by (E15), we have:

$$sgn \, dH/dw = sgn \left[\begin{aligned} & (w^*((1-\tau)(U' + (1-\tau)(1-H)U'')))a_{22} + \\ & -((1-\tau)(1-H)U''p)pa_{12} \end{aligned} \right] < 0; \tag{E22}$$

$$sgn \, dz/dw = sgn \left[\begin{aligned} & a_{11}((1-\tau)(1-H)U''p) + \\ & -a_{21}((1-\tau)(U' + (1-\tau)(1-H)w^*U'')) \end{aligned} \right] \geq 0; \tag{E23}$$

$$\text{sgn } dH/d\tau = \text{sgn} \begin{bmatrix} (-w^*(U' + w^*(1-\tau)(1-H)U'')) a_{22} + \\ -(-w^*(1-H)U'') p a_{12} \end{bmatrix} > 0; \quad (\text{E24})$$

$$\text{sgn } dz/d\tau = \text{sgn} \begin{bmatrix} a_{11}(-w^*(1-H)U''p) + \\ -a_{21}(-w^*(U' + w^*(1-\tau)(1-H)U'')) \end{bmatrix} \leq 0; \quad (\text{E25})$$

$$\begin{aligned} \text{sgn } dH/dT &= \text{sgn} \{ (1-\tau) w^* U'' (U'' p^2 + F'' + f'') - U'' p [(1-\tau) w^* p U'' + (F'' + f'') y_H] \} \\ &= \text{sgn} \{ U'' (F'' + f'') [(1-\tau) w^* - p y_H] \} = 0; \end{aligned} \quad (\text{E26})$$

$$\begin{aligned} \text{sgn } dz/dT &= \text{sgn} \left\{ \begin{aligned} &[(1-\tau)^2 (w^*)^2 U'' + (F'' + f'') (y_H)^2 + (F' + f') y_{HH}] U'' p + \\ &- (1-\tau) w^* U'' [(1-\tau) p w^* U'' + (F'' + f'') y_H] \end{aligned} \right\} = \\ &= \text{sgn} \left[\begin{aligned} &(U'')^2 (1-\tau)^2 (w^*)^2 p + U'' p (F'' + f'') (y_H)^2 + \\ &+ (F' + f') y_{HH} U'' p - (1-\tau)^2 (w^*)^2 (U'')^2 p - (1-\tau) w^* U'' (F'' + f'') y_H \end{aligned} \right] = \\ &= \text{sgn} [(F' + f') y_{HH} U'' p] \geq 0. \end{aligned} \quad (\text{E27})$$

Also, clearly, $dH/dE = 0$ and $dz/dE \leq 0$.

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